Collision Matrix for Low Earth Orbit Satellites

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The low Earth orbit (LEO) is becoming cluttered with thousands of satellites, rocket bodies, and a variety of space garbage. This collection of objects crossing paths at speeds on the order of 10 km/s is creating an increasing collision hazard to many operational systems. The effect that the destruction of LEO satellites will have on other users of the near-Earth environment is of great concern. A model is examined that quantifies the effect of one satellite's fragmentation on neighboring satellites. This model is used to evaluate the interdependent hazard to a series of satellite systems. A number of space system fragmentation events are numerically simulated and the collision hazard to each is tabulated. Once all satellites in the matrix have been fragmented separately, a complete collision hazard representation can be depicted. This model has potential for developing an enhanced understanding of a number of aspects of the growing debris hazard in LEO.

Introduction

THE international space launch rate has remained fairly constant, at about 120 annually, for the last 15 years. At the same time, the number of trackable objects in space has increased at nearly three times this rate.[1] Additionally, the trend for satellites is toward larger sizes and missions of longer duration. All these characteristics tend to produce an ever-increasing probability of collision for satellites in low Earth orbit (LEO).

Figure 1 is a depiction of the locations of objects in LEO at one point in time (July 1987). The plotting characters for these objects are not drawn to scale; the objects are much smaller in size in comparison to the Earth than the figure portrays. On the other hand, these satellites are moving at orbital velocities on the order of 7 km/s, resulting in average relative velocities between objects of about 10 km/s. The paths of these objects continually crisscross randomly in time and space.

The fragmentation of one of these satellites will affect many of its neighboring satellites. The initial velocities imparted to the fragments will produce a cloud of debris that will span a large range of inclinations, altitudes, and orbital periods. The variation in inclination values will be smaller in comparison to the others because it is more difficult to affect a change in the orientation of the orbit plane with respect to the Earth. For the same reason, inclination is relatively constant over a long period of time. Altitude variations may be great due to the relative ease of transferring energies within the orbital plane. The difference in orbital periods will cause the faster objects to overtake the slower objects over a period of time (i.e., make more revolutions).

The combination of these initial characteristics, coupled with secular perturbations, creates a complicated, time-varying hazard to other satellites. The effects from one breakup are far-reaching due to the dispersion of the debris in these three directions. The effect of the distribution of these objects is to produce varying spatial densities (objects per cubic kilometer). These variable spatial density values will in turn cause variations in the hazard to other satellites.

As more manned systems and multisatellite constellations are deployed, the need to understand the collateral impact of one on-orbit breakup event on other functioning satellites increases dramatically. A tabular method of assessing interdependent collision hazards is introduced. The collision matrix provides a quick-look representation of how selected satellite systems are affected by the background debris environment and the breakup of components of other satellite systems.

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SCREEN Program

The collision matrix combined multiple fragmentation scenarios using a model (SCREEN) developed in Ref. 2. The SCREEN program calculates the probability of collision for satellites at risk due to the debris generated by another satellite fragmenting. The debris is generated into three overlapping concentric debris clouds. The largest cloud is evenly distributed with millions of particles. The next smaller cloud is a factor of 10 smaller in size and is contained within the first cloud. The innermost cloud is another factor of 10 smaller, with thousands of fragments distributed evenly throughout it. This superposition produces a roughly exponential change in spatial densities going from the middle of the total cloud to the outer edge. (This general concept was adapted from the DEBRIS program developed by the Aerospace Corporation.) The hazard is found in three different phases, which parallel the evolution of the shape of the debris cloud.

Geometry

During the first phase the fragments are still configured in an elliptical cloud. The volume of the cloud used during all of phase 1 was carefully selected in Ref. 2. The volume chosen was that of the cloud after the center of mass had progressed one-quarter of a revolution after the breakup. During phase 1 the spatial density is the highest of the three phases, while the volume is the smallest.

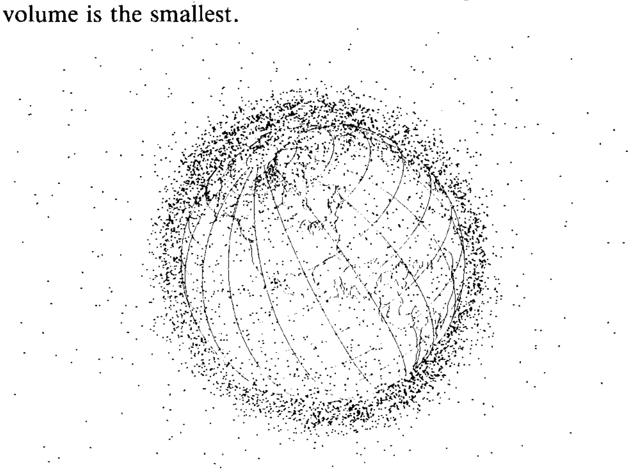

Fig. 1 A graphical representation of objects in orbit about the Earth (July 1987) provides a glimpse of the orbital debris hazard. Source: Ref. 1.

As time passes, the faster fragments overtake the slower fragments. Once the fastest fragment has caught up with the center of mass of the cloud, phase 1 is considered completed and phase 2 begins. It is important to note that the faster fragment merely completes one more revolution than the slower fragment; the two objects are not at the same altitude. During phase 2 the cloud is in the shape of the torus about the Earth. Initially, the torus is "pinched" at the breakup point and 180 deg away. This pinch is ignored since all the particles do not go through this point simultaneously and because this effect is short-lived. The cross section of the torus is determined by the extremes of altitude and inclination.

The variable regression of the right ascension of the debris fragments dismantles the torus over a few months. After a long period of time the cloud of debris forms a band about the Earth. It may take from about one year to as long as five years to achieve this final configuration, depending on the initial conditions at breakup.[3] The band is limited in latitude by the inclination of the parent satellite. Once the cloud has reached phase 3, the debris fragments become part of the background debris environment. The evolution of the debris cloud is shown in Fig. 2.

Probability of Collision

The probability of collision (PC) to a satellite at risk is calculated in each of the three phases. During the first two phases, a maximum probability (worst case) and an average probability are found. The maximum probability of collision during phase 1 (MAX PC1) is calculated by assuming that the satellite at risk passes through the largest dimension of the elliptical cloud possible. It is assumed that the orbital planes of the cloud and satellite at risk are perpendicular. This makes the relative velocity between the two 1.4 times their orbital velocities. The average probability of collision during phase 1 (AVE PC1) is found by multiplying MAX PC with the probability that the satellite at risk will encounter the debris cloud.

During phase 2, similar probabilities are calculated. MAX PC2 is the worst case probability in phase 2, while AVE PC2 is the average probability of collision. For both phases, the probability of collision values are the sum of the probability of collision with each of the three concentric debris volumes. The PC values during the first two phases are all for one pass through the cloud.

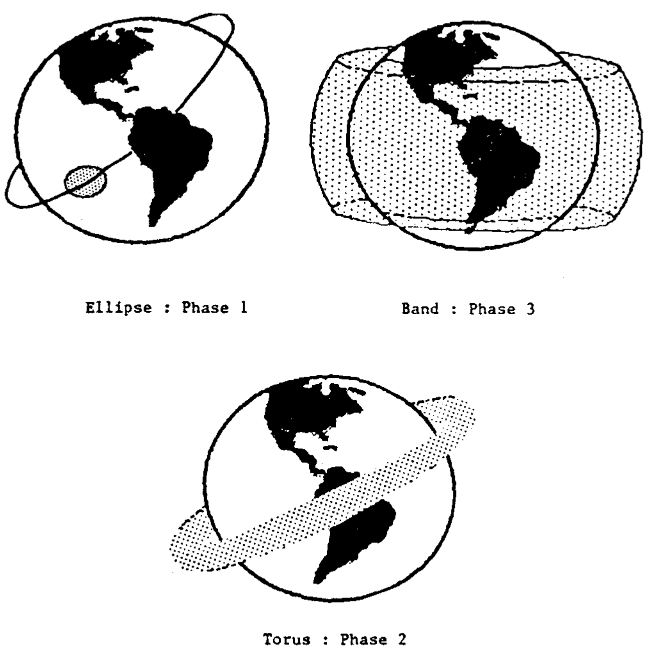

Fig. 2 It takes a number of years for the debris cloud to dismantle into a band about the Earth.

The PC during phase 3 is found by using spatial density values derived from the NORAD satellite catalog.[4] The fragments liberated by SCREEN are not included in this phase since most of them would not be trackable by NORAD. Additionally, by the time the cloud evolves into a band, a vast majority of these small objects will have re-entered the atmosphere.

A final PC term is calculated using a combination of the hazards during each of the three phases. The annual PC from the cloud (APC) is found by:

$$\begin{aligned} \text{APC} = &\ (\text{time in phase 1})\ (\text{AVE PC1}) \\ &+ (\text{time in phase 2})\ (\text{AVE PC2}) \\ &+ (\text{time left in one year}) \\ &\quad (\text{background debris hazard}) \end{aligned}$$

The "time left in one year" may be equal to zero if it takes more than one year for the cloud to develop into a band around the Earth. A sample listing is shown in Fig. 3.

Note that each PC value has four terms: one each for the three concentric clouds and their sum. It is the sum that is used in all the analyses. The individual cloud terms are of importance in determining the potential destructiveness of a possible collision. The objects in the inner cloud (third column) are considered to be larger and thus more destructive. These fragments may be in the centimeter to meter size range. Collision with these objects might result in major structural damage to the target satellite. The objects can be expected to get progressively smaller as the velocity imparted increases. The outer cloud (first column of numbers) represents micrometer-sized debris that would cause minimal damage to the target satellite even over a long period of time. It should be noted that this analysis does not account for re-entry of any of the generated debris fragments during the first two phases. This will cause the PC terms to be conservative. Because of the large uncertainty in the numbers of pieces originally produced, this simplification does not significantly affect the overall validity of the results.

Collision Matrix of LEO Satellites

The results from the iterative process of fragmentation and evaluation are contained in the collision matrix. Figure 4 outlines this process.

Each of a series of satellites is fragmented and the hazard from each event's debris to all the other satellites is evaluated. The fragmented satellites and the satellites at risk are the same. In Table 1, the file of these 20 satellite systems, BKP.DAT (fragmented satellites) and HAZ.DAT (satellites at risk) are listed.

The file identifies country of origin (C = 1 US, C = 2 USSR), the number of satellites in each system (N), name, inclination, apogee/perigee (km), and radar cross section (RCS in m^2). This list of systems is selected to provide a wide variety, but by no means is considered complete. The size and content of this matrix may vary depending on the needs of the user. The use of the collision matrix provides a tool to examine the interdependency of a satellite breakup on other operational satellites.

Example Collision Matrices

The collision matrix can help to highlight important characteristics of a well-traveled LEO environment. It is possible to

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SN12092 RCS 33.9251 AP/PH 1006/961
 MAX PC1 (PASS)   0.11E-1   0.47E-2   0.34E-3   **0.16E-1
 AVE PC1 (PASS)   0.92E-7   0.80E-7   0.57E-8   **0.18E-6
 MAX PC2 (PASS)   0.16E-4   0.67E-4   0.48E-4   **0.13E-3
 AVE PC2 (PASS)   0.92E-7   0.40E-7   0.29E-8   **0.14E-6
 ANNUAL PC FROM CLOUD (APC) 0.75E-4
 ANNUAL PC FROM BACKGROUND DEBRIS 0.87E-3
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Fig. 3 The PC values provide the hazard from the cloud across all phases of the cloud evolution.

Table 1 Satellites included in the collision matrix

No.	C	N	Name	Incl.	Apogee/Perigee, km	RCS, m^2
1	1	250	System 1 (US)	60.0	800/600	5.0
2	2	40	K1680, etc.	74.0	810/790	20.0
3	2	280	K1660, etc.	74.0	1550/1350	1.0
4	2	27	K1707, etc.	82.5	665/635	15.0
5	2	90	K1709, etc.	82.9	1020/965	20.0
6	2	35	Other	81.2	660/620	18.0
7	2	29	Other	81.2	900/800	15.0
8	2	1	MIR	51.6	450/180	275.0
9	1	4	NOAA 10	98.7	824/803	40.0
10	1	2	OSCAR 24	89.8	1258/1002	4.4
11	1	1	GEOSAT	8.0	785/783	31.9
12	1	1	ERBS	57.0	609/596	15.2
13	1	1	CCE	2.7	49700/1093	10.0
14	1	1	LDEF	28.5	473/471	45.2
15	1	2	LANDSATS	98.2	700/698	37.3
16	1	1	HILAT	82.0	834/765	5.2
17	1	1	NOAA 1	90.0	1192/1169	5.5
18	1	1	NOAA 6	98.5	813/797	28.5
19	1	1	NIMBUS 7	99.2	956/943	6.2
20	1	1	TIROS N	99.0	852/836	13.9

discriminate between the lethality of the different probabilities by identifying which of the three concentric debris clouds are encountered. The effect of varying the essential characteristics of the debris cloud (size and numbers of fragments) on other satellites is also examined. The collision matrix also provides a simple method for incorporating satellite growth rates into hazard models.

Simple Collision Matrix

Figures 5–9 are the collision matrices for the five PC terms discussed earlier. All the fragmentations had a maximum delta velocity of 2 km/s for the outer cloud. The two inner clouds were smaller by a factor of 10 and 100, respectively. The number of objects uniformly dispersed into each of these clouds are 2,600,000, 1,120,000, and 280,000 for outer to inner cloud.[3] The first column of all of the matrices (Annual Background) contains the annual PC for the satellite (system) at risk from the NORAD database of cataloged satellites. All the other probabilities are for the millions of fragments generated by a breakup (even though the vast majority of these would be too small to be tracked by NORAD). The hazard values represent base 10 exponents.

Any single hyphen in the matrices represents a zero probability of collision. It should also be noted that the "Ave" values shown are computed from the exact PC values (before rounding for inclusion into the matrix). For this reason the average of a row of numbers may not appear to match the average of the values shown. In discussing results from the collision matrices, FS4 will equate to "Fragmented Satellites is #4," while SR10 will represent "Satellite at Risk is #10." Therefore, "FS4/SR10" describes the situation where satellite #4 fragments and the PC for the satellite 10 is monitored.

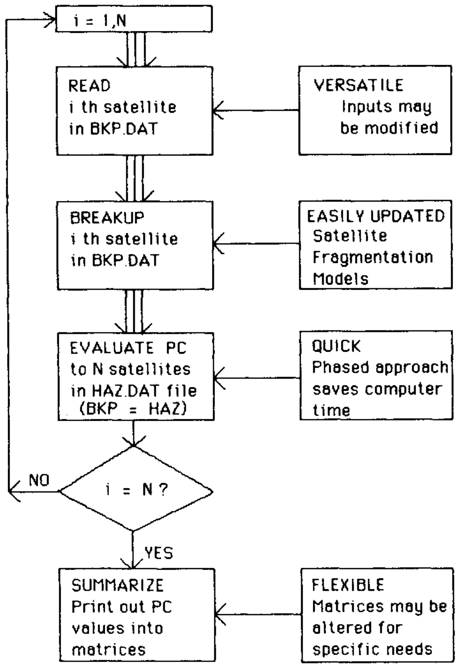

Fig. 4 The collision matrix was formulated to be versatile, easily updated, quick, and flexible.

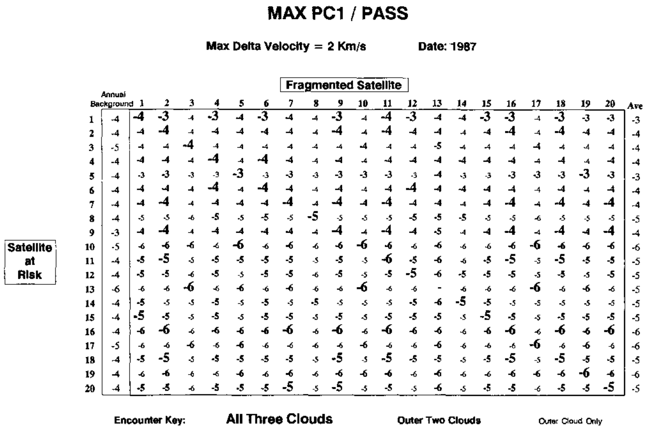

Fig. 5 The collision matrix for the MAX PC1 term with a maximum delta velocity of 2 km/s.

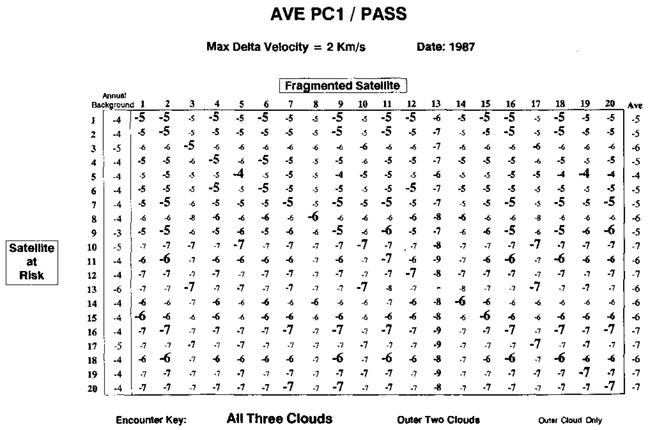

Fig. 6 The collision matrix for the AVE PC1 has values from 10 to 100 times smaller than the MAX PC1 matrix.

From these matrices a number of observations can be made. From Fig. 6, matrix of MAX PC1 values:

1) There is a clear diagonalization of the matrix when considering the type of encounter.

2) Satellites 1–6 are at high risk from almost any LEO satellite fragmenting. This is due mainly to the large number of satellites in each constellation.

3) MAX PC1/PASS values are within an order of magnitude of the average annual background debris hazard.

4) Satellites in constellations 1–7 and 9 are the most at risk.

From Fig. 6, matrix of AVE PC1 values, it can be seen that AVE PC1/PASS values are about 1–4 orders of magnitude smaller than the annual background debris hazard.

From Fig. 7, matrix of MAX PC2 values, it is clear that:

1) The MAX PC2 value is about 100 times greater than the AVE PC1 values.

2) The average effect of the breakup of fragmented satellites is nearly constant at 1E-4, the same as the averaged annual background hazard.

3) Satellites 1, 2, 4, 5, 6, and 7 are the most at risk.

Fig. 8, matrix of AVE PC2 values, shows that:

1) The AVE PC2 values are nearly identical to the AVE PC1 values. This similarity is due to a tradeoff between spatial density and the time of passage/probability of encountering the cloud. As we move from phase 1 to phase 2, the spatial density decreases but both the time of passage through cloud and probability of encountering cloud increases. The reason for this is discussed in detail in Ref. 2.

2) The breakup of satellite 13 continues to produce the least collateral impact on other satellites.

3) Satellites 1, 2, 4, 5, 6, and 7 are the most at risk.

It is obvious from Fig. 9, matrix of APC values, that:

1) All APC values are greater than or equal to the annual background hazard for each satellite at risk, except for satellite 13.

2) Satellites 1, 2, 4, 5, 6, and 7 are, again, the most at risk.

3) Excluding APC values from the breakup of satellite 13, combinations that create the smallest APC terms are FS3/SR8, (19), FS14/SR10, (12)(19), FS16/SR19, and almost all fragmentations for satellites 16–17 at risk.

On Figs. 5–9, an encounter key is located below the matrix. The inner cloud will contain the majority of the objects that have the potential of disrupting or even terminating the mission of a satellite. By noting this lethality measure, it may be said that:

1) Most probability of collision values involve encounters with the outermost two clouds. As such, these possible collisions would have only minimal impact on the operation of most satellite systems.

2) Satellite systems 1, 2, 7, 9, 16, and 18 have the most encounters with the inner cloud and are therefore the most at risk of being destroyed via collision.

3) There is a distinct diagonalization of the matrix when considering interaction with the innermost cloud.

In summary, satellite constellations 1, 2, 4, 5, 6, and 7 are the most susceptible to the breakup of other satellites included in the matrix. Additionally, even though the PC values for all phases are fairly randomly distributed, inclusion of the type of encounter shows a distinct diagonalization trend. This simply means that the breakup of one component of a satellite system provides a more lethal probability of collision to other member of the same constellation.

Vary Delta Velocity

Figures 10–12 are the collision matrices for a test run with the maximum delta velocity reduced by a factor of 10 (200 m/s). A separate analysis is not necessary for varying the number of objects (N) generated. A change in N would cause a proportional change in the PC values. A factor of 100 decrease in N would decrease the PC values by two orders of

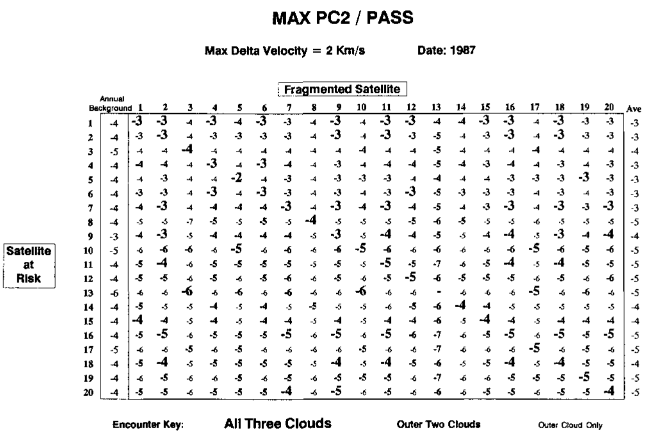

Fig. 7 The collision matrix for the MAX PC2 term with a maximum delta velocity of 2 km/s.

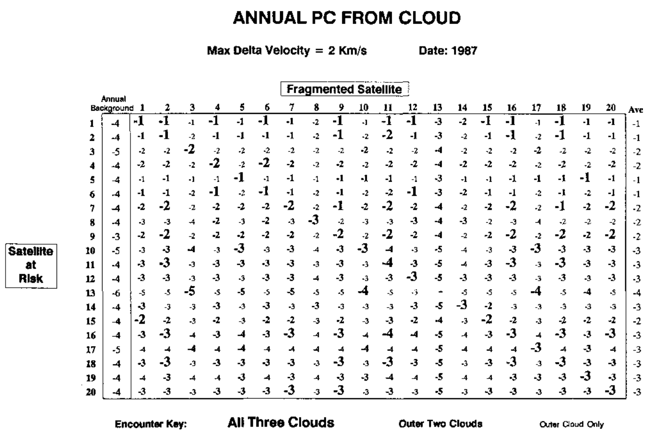

Fig. 9 The annual PC from cloud matrix displays the hazard for an entire year from the associated debris cloud.

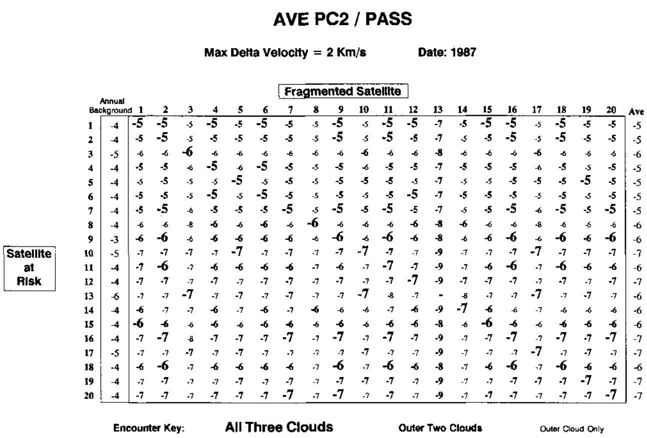

Fig. 8 The AVE PC2 matrix has values from 10 to 1000 times smaller than the MAX PC2 matrix.

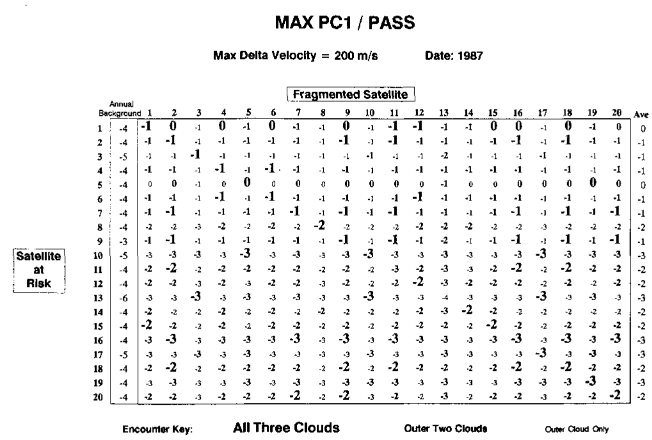

Fig. 10 For reduced delta velocity debris, assuming the same number of fragments, the MAX PC1 terms may increase drastically.

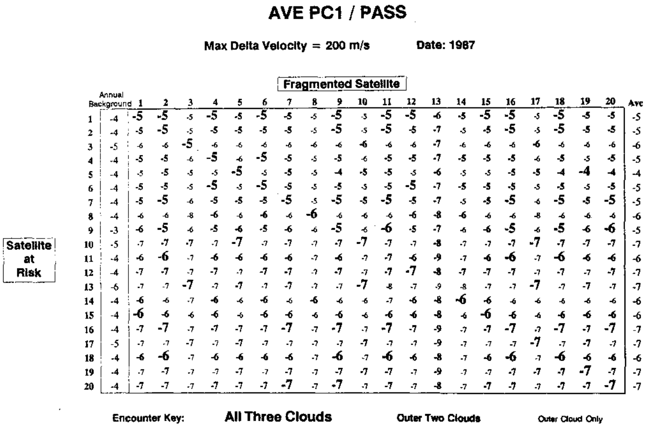

Fig. 11 The AVE PC1 matrix is not affected by the reduced delta velocity of fragments.

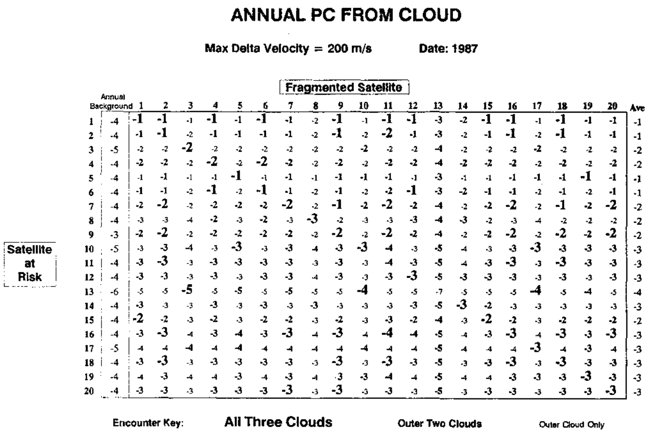

Fig. 12 The reduced delta velocity fragmentation produces a similar hazard over a year's time.

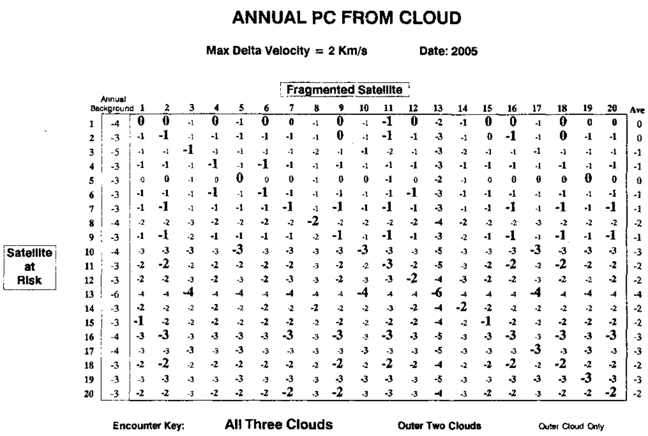

Fig. 13 Projecting the growth of satellite constellations shows that the annual hazard may be much greater by the year 2005.

magnitude across all values. A change in delta velocity does not cause such a predictable or linear effect.

From these figures a number of observations are made:

1) The PC values are lower across the three matrices by one to nine orders of magnitude vs Figs. 5–9.

2) The smaller delta velocity (thus smaller initial cloud) resulted in much larger variations in hazard: a hit-or-miss proposition. So a smaller magnitude breakup does not necessarily create a smaller hazard, it may just create a larger hazard for fewer satellites.

3) By changing the fragmentation model slightly, drastic variations in the satellites at risk and the magnitude of that risk resulted. The PC values from the lower delta velocity clouds are about 1000 times larger than the PC values for the larger delta velocities. This re-emphasizes the need for a better model to describe the fragmentation process.

4) A slight diagonalization of the matrix has been noted for PC values and encounter cases. That is to say, the breakup of one component of a constellation affects the other components significantly. This trend was existent in the larger delta velocity breakups when encounter scenarios were considered.

Future Growth

The satellite population continues to grow, which in turn increases the hazard to operational satellites. The "health and well-being" of all satellites in LEO affect many others as shown by the collision matrix. This tool may also be used to determine how satellite growth will impact future satellite operations.

Increasing the background spatial density values by 10% annually can be incorporated into the model. Additionally, the number of satellites in each constellation can be increased according to some annual replacement rate. Let us use a net 5% replacement rate for each constellation. Using both of these growth rates, the APC collision matrix for the year 2005 follows (Fig. 13). This matrix provides valuable insight into the future collision hazard to specific satellite systems and the general condition of LEO, with:

1) The annual background hazard will go up by an order of magnitude for 14 of the 20 satellite constellations by 2005. This assumes that there are no explosions or collision-induced breakups in LEO over this time.

2) About 75% of the 400 values in the core of the APC matrix will go up by a factor of 10 by 2005. The average hazard for each satellite system (last column) increased by one order of magnitude for 12 of these systems (satellites 1, 2, 3, 4, 5, 7, 9, 11, 12, 14, 18, and 20).

3) The complicated LEO environment can be monitored in the long term through the use of the collision matrix.

Summary

It is evident that the breakup of a satellite will have far-reaching effects by increasing the hazard to many other satellite constellations. This interdependency is quickly depicted in both the short term and long term by the use of the collision matrix. It is shown that a larger magnitude breakup is not necessarily more hazardous to other LEO satellites. A more energetic fragmentation will distribute debris over a larger range of altitudes, affecting more satellites but to a lesser degree since lower spatial density values will result. For a less energetic breakup, the cloud will be more compact, creating a hazard to fewer satellites but the hazard will be much greater. This trend especially affects component-component hazards, causing a diagonalization of the collision matrix.

Selected growth rates of the background environment and satellite constellation size result in significant hazard increases by the year 2005. The collision matrix provides a flexible, easy-to-use tool to track changes in the long-term hazard to LEO satellites. Specific user needs can easily be incorporated into this program: fragmentation model, satellites under analysis, and timing of cloud phases.

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